

# Cosmology in presence of dark energy in an emergent gravity scenario

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(Dated: February 24, 2015)

We obtain the analogues of the Friedman equations in an emergent gravity scenario in the presence of dark energy. The background metric is taken to be Friedman-Lemaitre-Robertson-Walker (FLRW). We show that if  $\dot{\phi}^2$  is the dark energy density (in units of the critical density) then (a) for total energy density greater than the pressure (non-relativistic scenario, matter domination) the deceleration parameter  $q(t) \approx \frac{1}{2}[1 + 27\dot{\phi}^2 + \dots] > \frac{1}{2}$  (b) for total energy density equal to 3 times the pressure (relativistic case, radiation domination), the deceleration parameter  $q(t) \approx 1 + 18\dot{\phi}^2 + \dots > 1$  and (c) for total energy density equal to the negative of the pressure (dark energy scenario), the deceleration parameter  $q(t) < -1$ . Our results indicate that many aspects of standard cosmology can be accommodated with the presence of dark energy right from the beginning of the universe where the time parameter  $t \equiv \frac{t}{t_0}$ ,  $t_0$  being the present epoch.

PACS numbers: 98.80.-k ; 95.36.+x, 04.20.-q

## INTRODUCTION

Scalar fields  $\phi$  whose lagrangians are non-canonical are candidates for  $k$ -essence fields that give rise to dark energy. The general form for such lagrangians is proportional to  $F(X)$  with  $X = \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$ . Relevant literature for such fields in cosmology, inflation, dark matter, dark energy and strings can be found in [1]-[17].

A natural question is whether the standard cosmology is modified if we take into account the presence of dark energy while building up the Friedman equations. In this context it has to be further remembered that dynamical solutions of the  $k$ -essence equation of motion changes the metric for the perturbations around these solutions [18]-[22]. The perturbations propagate in an emergent spacetime with metric  $\tilde{G}^{\mu\nu}$  different from (and also not conformally equivalent to) the gravitational metric  $g^{\mu\nu}$ .  $\tilde{G}^{\mu\nu}$  now depends on  $\phi$ . Hence we have to determine the analogue of the Friedman equations in the context of dark energy in an emergent gravity scenario.

The motivation of this work is to seek plausible answers to the above question. Taking the background metric to be FLRW, we obtain the modifications of the standard cosmological parameters in the radiation dominated, matter dominated and dark energy dominated phases of the universe. The dark energy density is identified with the kinetic energy  $\dot{\phi}^2$  of the  $k$ -essence field. The standard cosmological parameters are retrieved when  $\dot{\phi}^2 \rightarrow 0$ , i.e., the dark energy vanishes.

The plan of the paper is as follows: section 2 introduces emergent gravity concepts, section 3 discusses the emergent gravity equations of motion, section 4 contains the analogues of the Friedman equations in the presence of dark energy, section 5 discusses the solutions of the resulting cosmological equations arising from emergent gravity equations of motion and the analogue of the

Friedman equations and section 6 comprises of our conclusions.

## EMERGENT GRAVITY

The  $k$ -essence scalar field  $\phi$  is minimally coupled to the gravitational field  $g_{\mu\nu}$  and the action is:

$$S_k[\phi, g_{\mu\nu}] = \int d^4x \sqrt{-g} L(X, \phi) \quad (1)$$

where  $X = \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$  and the energy momentum tensor for the  $k$ -essence scalar field is:

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta S_k}{\delta g^{\mu\nu}} = L_X \nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}L \quad (2)$$

$L_X = \frac{dL}{dX}$ ,  $L_{XX} = \frac{d^2L}{dX^2}$ ,  $L_\phi = \frac{dL}{d\phi}$  and  $\nabla_\mu$  is the covariant derivative defined with respect to the metric  $g_{\mu\nu}$ . The equation of motion for the scalar field is [23]:

$$-\frac{1}{\sqrt{-g}} \frac{\delta S_k}{\delta \phi} = \tilde{G}^{\mu\nu} \nabla_\mu \nabla_\nu \phi + 2X L_{X\phi} - L_\phi = 0 \quad (3)$$

where the effective metric  $\tilde{G}^{\mu\nu}$  is

$$\tilde{G}^{\mu\nu}(\phi, \nabla\phi) \equiv L_X g^{\mu\nu} + L_{XX} \nabla^\mu\phi \nabla^\nu\phi \quad (4)$$

and is physically meaningful only when  $1 + \frac{2X L_{XX}}{L_X} > 0$ .

We first carry out the conformal transformation  $G^{\mu\nu} \equiv \frac{c_s}{L_X^2} \tilde{G}^{\mu\nu}$ , with  $c_s^2(X, \phi) \equiv (1 + 2X \frac{L_{XX}}{L_X})^{-1} \equiv \text{sound speed}$ . Then the inverse metric of  $G^{\mu\nu}$  is

$$G_{\mu\nu} = \frac{L_X}{c_s} [g_{\mu\nu} - c_s^2 \frac{L_{XX}}{L_X} \nabla_\mu\phi \nabla_\nu\phi] \quad (5)$$

A further conformal transformation  $\bar{G}_{\mu\nu} \equiv \frac{c_s}{L_X} G_{\mu\nu}$  gives

$$\bar{G}_{\mu\nu} = g_{\mu\nu} - \frac{L_{XX}}{L_X + 2X L_{XX}} \nabla_\mu\phi \nabla_\nu\phi \quad (6)$$

$L_X \neq 0$  always for the sound speed  $c_s^2$  to be positive definite and only then equations (1) – (4) is physically meaningful. This can be seen as follows.  $L_X = 0$  implies that  $L$  does not depend on  $X$  so that in equation (1),  $L(X, \phi) \equiv L(\phi)$ . So the  $k$ -essence lagrangian  $L$  becomes pure potential. Then the very definition of  $k$ -essence fields is meaningless because such fields correspond to lagrangians where the kinetic energy dominates over the potential energy. So lagrangian cannot be pure potential. Also, the very concept of minimally coupling the  $k$ -essence field  $\phi$  to the gravitational field  $g_{\mu\nu}$  becomes redundant and equation (1) meaningless and equations (4-6) ambiguous.

Non-trivial configurations imply  $\partial_\mu \phi \neq 0$  and  $\bar{G}_{\mu\nu}$  not conformally equivalent to  $g_{\mu\nu}$ . So this  $k$ -essence field has properties different from canonical scalar fields defined with  $g_{\mu\nu}$  and the local causal structure is also different from those defined with  $g_{\mu\nu}$ . Further, if  $L$  is not an explicit function of  $\phi$  then equation (3) becomes ;

$$-\frac{1}{\sqrt{-g}} \frac{\delta S_k}{\delta \phi} = \bar{G}^{\mu\nu} \nabla_\mu \nabla_\nu \phi = 0 \quad (7)$$

We take  $L = L(X) = 1 - V\sqrt{1 - 2X}$  [24]. This is a particular case of the Born-Infeld lagrangian  $L(X, \phi) = 1 - V(\phi)\sqrt{1 - 2X}$  for  $V(\phi) = V = \text{constant}$  and  $V \ll \text{kinetic energy of } \phi$  i.e.  $V \ll (\dot{\phi})^2$ . Then  $c_s^2(X, \phi) = 1 - 2X$ . For scalar fields  $\nabla_\mu \phi = \partial_\mu \phi$ . So (6) becomes

$$\bar{G}_{\mu\nu} = g_{\mu\nu} - \partial_\mu \phi \partial_\nu \phi \quad (8)$$

Note that the first conformal transformation is used to identify the inverse metric  $G_{\mu\nu}$ . The second conformal transformation realises the mapping onto the metric given in (8) for  $L(X) = 1 - V\sqrt{1 - 2X}$ .

Consider the second conformal transformation  $\bar{G}_{\mu\nu} \equiv \frac{c_s}{L_X} G_{\mu\nu}$ . Following [18] the new Christoffel symbols are related to the old ones by

$$\begin{aligned} \bar{\Gamma}_{\mu\nu}^\alpha &= \Gamma_{\mu\nu}^\alpha + (1 - 2X)^{-1/2} G^{\alpha\gamma} [G_{\mu\gamma} \partial_\nu (1 - 2X)^{1/2} \\ &\quad + G_{\nu\gamma} \partial_\mu (1 - 2X)^{1/2} - G_{\mu\nu} \partial_\gamma (1 - 2X)^{1/2}] \\ &= \Gamma_{\mu\nu}^\alpha - \frac{1}{2(1 - 2X)} [\delta_\mu^\alpha \partial_\nu X + \delta_\nu^\alpha \partial_\mu X] \end{aligned} \quad (9)$$

The second term on the right hand side is symmetric under exchange of  $\mu$  and  $\nu$  so that the symmetry of  $\bar{\Gamma}$  is maintained. The second term originates from the  $k$ -essence lagrangian and this additional term signifies additional interactions (forces). The new geodesic equation in terms of  $\bar{\Gamma}$  now is

$$\frac{d^2 x^\alpha}{d\tau^2} + \bar{\Gamma}_{\mu\nu}^\alpha \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0 \quad (10)$$

### EMERGENT EQUATIONS OF MOTION FOR FLRW GRAVITATIONAL METRIC

Take the gravitational metric  $g_{\mu\nu}$  to be FLRW and assume that the  $k$ -essence scalar field  $\phi(r, t)$  is spherically

symmetric ( $\partial_t \phi = \partial_0 \phi = \dot{\phi}$  and  $\partial_r \phi = \partial_1 \phi = \phi'$ ). Then (8) becomes

$$\begin{aligned} \bar{G}_{00} &= g_{00} - (\partial_0 \phi)^2 = 1 - \dot{\phi}^2 \\ \bar{G}_{11} &= g_{11} - (\partial_r \phi)^2 = -\frac{a^2(t)}{1 - Kr^2} - (\phi')^2 \\ \bar{G}_{22} &= g_{22} = -a^2(t)r^2 \\ \bar{G}_{33} &= g_{33} = -a^2(t)r^2 \sin^2 \theta \\ \bar{G}_{01} &= \bar{G}_{10} = -\dot{\phi}\phi' \end{aligned} \quad (11)$$

where the FLRW metric components are  $g_{00} = 1; g_{11} = -\frac{a^2(t)}{1 - Kr^2}; g_{22} = -a^2(t)r^2; g_{33} = -a^2(t)r^2 \sin^2 \theta; g_{ij} (i \neq j) = 0$ .

The line element becomes

$$ds^2 = (1 - \dot{\phi}^2)dt^2 - \left( \frac{a^2}{1 - Kr^2} + (\phi')^2 \right) dr^2 - 2\dot{\phi}\phi' dt dr - a^2 r^2 d\Omega^2 \quad (12)$$

with  $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ .

Consider a co-ordinate transformation from  $(t, r, \theta, \phi)$  to  $(\omega, r, \theta, \phi)$  so that [25]:

$$d\omega = dt - \left( \frac{\dot{\phi}\phi'}{1 - \dot{\phi}^2} \right) dr \quad (13)$$

Then (12) becomes

$$ds^2 = (1 - \dot{\phi}^2)d\omega^2 - \left[ \frac{a^2}{1 - Kr^2} + (\phi')^2 + \frac{(\dot{\phi}\phi')^2}{(1 - \dot{\phi}^2)} \right] dr^2 - a^2 r^2 d\Omega^2 \quad (14)$$

i.e.

$$\bar{G}_{\mu\nu} = \begin{pmatrix} (1 - \dot{\phi}^2) & 0 & 0 & 0 \\ 0 & -Z & 0 & 0 \\ 0 & 0 & -(a^2 r^2) & 0 \\ 0 & 0 & 0 & -(a^2 r^2 \sin^2 \theta) \end{pmatrix} \quad (15)$$

while its inverse is

$$\bar{G}^{\mu\nu} = \begin{pmatrix} (1 - \dot{\phi}^2)^{-1} & 0 & 0 & 0 \\ k0 & -Z^{-1} & 0 & 0 \\ 0 & 0 & -(a^2 r^2)^{-1} & 0 \\ 0 & 0 & 0 & -(a^2 r^2 \sin^2 \theta)^{-1} \end{pmatrix} \quad (16)$$

with  $Z = \left( \frac{a^2}{1 - Kr^2} + (\phi')^2 + \frac{(\dot{\phi}\phi')^2}{(1 - \dot{\phi}^2)} \right)$ .

The equation (7) means

$$\begin{aligned} \bar{G}^{00} \partial_0^2 \phi + \bar{G}^{11} (\partial_1^2 \phi - \frac{a\dot{a}}{1 - Kr^2} \partial_0 \phi \\ - \frac{Kr}{1 - Kr^2} \partial_1 \phi) = 0 \end{aligned} \quad (17)$$

i.e.

$$\begin{aligned} & \ddot{\phi}[a^2(1-\dot{\phi}^2) + (\phi')^2(1-Kr^2)] \\ &= (1-\dot{\phi}^2)^2[\phi''(1-Kr^2) - a\dot{a}\dot{\phi} - Kr\phi'] \end{aligned} \quad (18)$$

We shall, henceforth, consider the FLRW universe for homogeneous dark energy fields only. So

$$\phi(r, t) \equiv \phi(t) \quad (19)$$

Here  $\dot{\phi}^2 \neq 0$  since the  $k$ -essence field must have non-zero kinetic energy. Also  $\dot{\phi}^2 \neq 1$  because  $\Omega_{matter} + \Omega_{radiation} + \Omega_{darkenergy} = 1$  and  $\dot{\phi}^2$  measured in units of the critical density is nothing but  $\Omega_{darkenergy}$ . Further,  $\dot{\phi}^2 < 1$  always in order that the signature of the metric (15) does not become ill-defined. Therefore  $0 < \dot{\phi}^2 < 1$ . Therefore (18) becomes

$$\frac{\dot{a}}{a} = H(t) = -\frac{\ddot{\phi}}{\dot{\phi}(1-\dot{\phi}^2)} \quad (20)$$

where  $H(t) = \frac{\dot{a}}{a}$  is Hubble parameter (*always*  $\dot{a} \neq 0$ ). So the equations of motion of emergent gravity relate the Hubble parameter to time derivatives of the  $k$ -essence scalar field.

### THE ANALOGUE OF FRIEDMANN EQUATIONS IN PRESENCE OF DARK ENERGY

Using metrics (15 and 16) for homogeneous fields  $\phi(t)$  we get the non-vanishing connection coefficients as:

$$\Gamma_{00}^0 = -\frac{\dot{\phi}\ddot{\phi}}{1-\dot{\phi}^2};$$

$$\bar{\Gamma}_{11}^0 = \frac{1}{1-\dot{\phi}^2} \frac{a\dot{a}}{1-Kr^2};$$

$$\bar{\Gamma}_{22}^0 = \frac{a\dot{a}r^2}{1-\dot{\phi}^2};$$

$$\bar{\Gamma}_{33}^0 = \frac{a\dot{a}r^2 \sin^2\theta}{1-\dot{\phi}^2};$$

$$\bar{\Gamma}_{01}^1 = \bar{\Gamma}_{10}^1 = \frac{\dot{a}}{a};$$

$$\bar{\Gamma}_{11}^1 = \frac{Kr}{1-Kr^2};$$

$$\bar{\Gamma}_{22}^1 = -r(1-Kr^2);$$

$$\bar{\Gamma}_{33}^1 = -r \sin^2\theta(1-Kr^2);$$

$$\bar{\Gamma}_{02}^2 = \bar{\Gamma}_{20}^2 = \frac{\dot{a}}{a};$$

$$\bar{\Gamma}_{12}^2 = \bar{\Gamma}_{21}^2 = \frac{1}{r};$$

$$\bar{\Gamma}_{33}^2 = -\sin\theta \cos\theta;$$

$$\bar{\Gamma}_{03}^3 = \bar{\Gamma}_{30}^3 = \frac{\dot{a}}{a};$$

$$\bar{\Gamma}_{13}^3 = \bar{\Gamma}_{31}^3 = \frac{1}{r};$$

$$\bar{\Gamma}_{23}^3 = \bar{\Gamma}_{32}^3 = \cot\theta.$$

Now we calculate the diagonal components of Ricci tensor for homogeneous scalar field since off-diagonal components of Ricci tensor are zero.

$$\bar{R}_{00} = 3\frac{\ddot{a}}{a} + 3\frac{\dot{a}}{a} \frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)} \quad (21)$$

$$\begin{aligned} \bar{R}_{11} = & -\frac{a^2}{1-Kr^2} \left[ \frac{\ddot{a}}{a} \frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}^2}{a^2} \frac{1}{(1-\dot{\phi}^2)} \right. \\ & \left. + 2\frac{K}{a^2} + \frac{\dot{a}}{a} \frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2} \right] \end{aligned} \quad (22)$$

$$\begin{aligned} \bar{R}_{22} = & -a^2 r^2 \left[ \frac{\ddot{a}}{a} \frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}^2}{a^2} \frac{1}{(1-\dot{\phi}^2)} + 2\frac{K}{a^2} \right. \\ & \left. + \frac{\dot{a}}{a} \frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2} \right] \end{aligned} \quad (23)$$

$$\begin{aligned} \bar{R}_{33} = & -a^2 r^2 \sin^2\theta \left[ \frac{\ddot{a}}{a} \frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}^2}{a^2} \frac{1}{(1-\dot{\phi}^2)} \right. \\ & \left. + 2\frac{K}{a^2} + \frac{\dot{a}}{a} \frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2} \right] \end{aligned} \quad (24)$$

To calculate Ricci scalar:

$$\bar{R}_0^0 = \bar{G}^{00} \bar{R}_{00} = \frac{1}{(1-\dot{\phi}^2)} \left[ 3\frac{\ddot{a}}{a} + 3\frac{\dot{a}}{a} \frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)} \right] \quad (25)$$

$$\begin{aligned} \bar{R}_1^1 = \bar{G}^{11} \bar{R}_{11} = & \left[ \frac{\ddot{a}}{a} \frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}^2}{a^2} \frac{1}{(1-\dot{\phi}^2)} \right. \\ & \left. + 2\frac{K}{a^2} + \frac{\dot{a}}{a} \frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2} \right] \end{aligned} \quad (26)$$

$$\begin{aligned}\bar{R}_2^2 = \bar{G}^{22} \bar{R}_{22} = & \left[ \frac{\ddot{a}}{a} \frac{1}{(1 - \dot{\phi}^2)} + 2 \frac{\dot{a}^2}{a^2} \frac{1}{(1 - \dot{\phi}^2)} \right. \\ & \left. + 2 \frac{K}{a^2} + \frac{\dot{a}}{a} \frac{\dot{\phi} \ddot{\phi}}{(1 - \dot{\phi}^2)^2} \right]\end{aligned}\quad (27)$$

$$\begin{aligned}\bar{R}_3^3 = \bar{G}^{33} \bar{R}_{33} = & \left[ \frac{\ddot{a}}{a} \frac{1}{(1 - \dot{\phi}^2)} + 2 \frac{\dot{a}^2}{a^2} \frac{1}{(1 - \dot{\phi}^2)} \right. \\ & \left. + 2 \frac{K}{a^2} + \frac{\dot{a}}{a} \frac{\dot{\phi} \ddot{\phi}}{(1 - \dot{\phi}^2)^2} \right]\end{aligned}\quad (28)$$

Therefore the Ricci Scalar:

$$\begin{aligned}\bar{R} = \bar{R}_0^0 + \bar{R}_1^1 + \bar{R}_2^2 + \bar{R}_3^3 \\ = 6 \left[ \frac{\ddot{a}}{a} \frac{1}{(1 - \dot{\phi}^2)} + \frac{\dot{a}^2}{a^2} \frac{1}{(1 - \dot{\phi}^2)} + \frac{K}{a^2} \right. \\ \left. + \frac{\dot{a}}{a} \frac{\dot{\phi} \ddot{\phi}}{(1 - \dot{\phi}^2)^2} \right]\end{aligned}\quad (29)$$

We have the Einstein's Field Equation:  $\bar{E}_{\mu\nu} = \bar{R}_{\mu\nu} - \frac{1}{2} \bar{G}_{\mu\nu} \bar{R} = -8\pi G T_{\mu\nu}$  i.e.

$$\bar{E}_\mu^\nu = \bar{R}_\mu^\nu - \frac{1}{2} \delta_\mu^\nu \bar{R} = -8\pi G T_\mu^\nu \quad (30)$$

where  $G$  is gravitational constant and  $T_{\mu\nu}$  is energy-momentum tensor. Components of Einstein tensor are:

$$\bar{E}_0^0 = -3 \left[ \frac{\dot{a}^2}{a^2} \frac{1}{(1 - \dot{\phi}^2)} + \frac{K}{a^2} \right]; \quad (31)$$

$$\begin{aligned}\bar{E}_1^1 = \bar{E}_2^2 = \bar{E}_3^3 = & -2 \left[ \frac{\ddot{a}}{a} \frac{1}{(1 - \dot{\phi}^2)} + \frac{\dot{a}^2}{a^2} \frac{1}{(1 - \dot{\phi}^2)} \right. \\ & \left. + \frac{K}{a^2} + 2 \frac{\dot{a}}{a} \frac{\dot{\phi} \ddot{\phi}}{(1 - \dot{\phi}^2)^2} \right]\end{aligned}\quad (32)$$

The energy-momentum tensor of an ideal fluid is

$$T_\mu^\nu = (p + \rho) u_\mu u^\nu - \delta_\mu^\nu p \quad (33)$$

where  $p$  is pressure and  $\rho$  is the matter density of the cosmic fluid. In the co-moving frame we have  $u^0 = 1$  and  $u^\alpha = 0$ ;  $\alpha = 1, 2, 3$ .

Now the general  $k$ -essence field theoretic lagrangian  $L(X, \phi)$ , which explicitly depends on  $\phi$ , is not equivalent to isentropic hydrodynamics because  $\phi$  and  $X$  are independent and hence the pressure cannot be a function of the energy density  $\rho$  only. So a pertinent question is whether we are at all justified in assuming a perfect fluid model when dark energy is present. The answer is yes because our lagrangian  $L(X) = 1 - V\sqrt{1 - 2X}$ , where  $V$  is a constant, does not depend explicitly on  $\phi$ . This class of models is equivalent to perfect fluid models with zero

vorticity and the pressure (lagrangian) can be expressed through the energy density only [23].

Then (33) becomes

$$T_0^0 = \rho; \quad T_1^1 = T_2^2 = T_3^3 = -p \quad (34)$$

Using equations (30)-(33) we get

$$\rho_d = \frac{3}{8\pi G} \left[ \frac{\dot{a}^2}{a^2} \frac{1}{(1 - \dot{\phi}^2)} + \frac{K}{a^2} \right] \quad (35)$$

and

$$\begin{aligned}p_d = & -\frac{1}{8\pi G} \left[ 2 \frac{\ddot{a}}{a} \frac{1}{(1 - \dot{\phi}^2)} + \frac{\dot{a}^2}{a^2} \frac{1}{(1 - \dot{\phi}^2)} + \frac{K}{a^2} \right. \\ & \left. + 2 \frac{\dot{a}}{a} \frac{\dot{\phi} \ddot{\phi}}{(1 - \dot{\phi}^2)^2} \right]\end{aligned}\quad (36)$$

where we now replace  $\rho$  by  $\rho_d$  as the total matter density in presence of dark energy and  $p$  by  $p_d$  as the pressure when dark energy is present. Note that both  $\rho_d, p_d$  reduce to the usual quantities  $\rho, p$  [26, 27] when dark energy is absent, i.e.,  $(\dot{\phi})^2 = 0$ . The usual Friedman equations are now modified into the above two equations (35 and 36) in the presence of  $k$ -essence scalar field  $\phi$ .

Combining above two equations (35 and 36) we get,

$$\frac{4\pi G}{3} (\rho_d + 3p_d) = - \left[ \frac{\ddot{a}}{a} \frac{1}{(1 - \dot{\phi}^2)} + \frac{\dot{a}}{a} \frac{\dot{\phi} \ddot{\phi}}{(1 - \dot{\phi}^2)^2} \right] \quad (37)$$

Now differentiating equation (35) with respect to cosmic time  $t$  and substituting the result in equation (37) we get,

$$\dot{\rho}_d = -3 \frac{\dot{a}}{a} (\rho_d + p_d) = -3H(\rho_d + p_d) \quad (38)$$

which is the required energy conservation equation in presence of dark energy. Again it may be noted that one recovers the usual energy conservation equation [26, 27] when dark energy is absent.

Now assume that the criterion for non-relativistic scenario remains the same, *viz.*,  $\rho_d \gg p_d$ . We restrict now to  $K = 0$  as observationally this is most likely. Then the above condition reduces to  $\frac{\dot{a}^2}{a^2} \gg -\frac{\ddot{a}}{2a} - \frac{\dot{a}\dot{\phi}\ddot{\phi}}{2a(1-\dot{\phi}^2)}$ . So the second term on right hand side must be always positive i.e.  $\dot{\phi}\ddot{\phi}$  must be always negative. This means that  $\frac{d\dot{\phi}^2}{dt} < 0$ . This criterion is consistent with the fact that the dark energy density cannot increase in a matter dominated era.

Then, neglecting  $p_d$  in (38) gives  $\dot{\rho}_d \frac{a}{\rho_d} + 3\rho_d = 0$  which has the solution

$$\rho_d^{mat} = \frac{A}{a^3} \quad (39)$$

where  $A = \text{constant}$ . Assuming that the total energy i.e.  $\rho_d a^3$  is a constant, we equate this to the present epoch energy i.e.  $\rho_d a^3 = \rho_{d0} a_0^3$ , where  $\rho_{d0}$  and  $a_0$  are

the matter density and scale radius at the present epoch ( $t = t_0$ ). This fixes the constant  $A = \rho_{d0} a_0^3$  in terms of present epoch values.

For the relativistic situation we assume again that the criterion is same as in standard cosmology, i.e.  $p_d = \frac{\rho_d}{3}$ . Here this gives the condition  $\frac{\dot{a}^2}{a^2} = -\frac{\ddot{a}}{a} - \frac{\dot{a}\ddot{\phi}}{a(1-\dot{\phi}^2)}$ . For same reasons as given in the previous case, here also the conditions are consistent.

We get from (38) the solution

$$\rho_d^{rad} = \frac{B}{a^4} \quad (40)$$

where the constant  $B$  is fixed to be  $B = \rho_{d0} a_0^4$  following same arguments as before.

Finally we consider the dark energy dominated scenario  $p_d = -\rho_d$ , i.e.,  $\frac{\dot{a}^2}{a^2} = \frac{\ddot{a}}{a} + \frac{\dot{a}\ddot{\phi}}{a(1-\dot{\phi}^2)}$ . This means that here  $\frac{d\dot{\phi}^2}{dt} > 0$  i.e. the dark energy density must increase. This is also consistent. Equation (38) then leads to

$$\rho_d = W \quad (41)$$

where we may choose the constant  $W$  to be  $\dot{\phi}^2|_{t=t'}$  with  $t'$  denoting some specific epoch.

Therefore, the difference from the standard cosmology lies only in the fact that in our case the dark energy density (which is being identified with the kinetic energy of the  $k$ -essence field) has the following behaviour: *in the matter and radiation dominated eras the time rate of change of dark energy density decreases, while in the dark energy dominated epoch this rate increases.*

## SOLUTIONS OF THE MODIFIED EQUATIONS

### Non-relativistic case (Matter dominated Universe)

The non-relativistic case means  $\rho_d \gg p_d$  and (35) and (36) can be written as follows,

$$\frac{H^2}{(1-\dot{\phi}^2)} + \frac{K}{a^2} = \frac{8\pi G}{3} \frac{A}{a^3} \quad (42)$$

and

$$2\frac{\ddot{a}}{a} \frac{1}{(1-\dot{\phi}^2)} + \frac{H^2}{(1-\dot{\phi}^2)} + \frac{K}{a^2} + 2H \frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2} = 0 \quad (43)$$

Eliminating  $A$  using (39), and remembering that for  $t = t_0$  (present epoch)  $\rho_d = \rho_{d0}$ ,  $a = a_0$ ,  $H = H_0$  (42) becomes

$$\frac{K}{a_0^2} = \frac{8\pi G}{3} [\rho_{d0} - \rho_d^c] \quad (44)$$

where

$$\rho_d^c = \frac{3H_0^2}{8\pi G(1-\dot{\phi}^2)} \quad (45)$$

is the critical value of matter density when dark energy is present. For  $\dot{\phi}^2 < 1$  (45) becomes

$$\rho_d^c = \rho_c + \rho_c \dot{\phi}^2 \quad (46)$$

keeping terms upto  $O(\dot{\phi}^2)$  only. Here  $\rho_c = \frac{3H_0^2}{8\pi G}$ , the critical value of matter density and  $\rho_d^c > \rho_c$ .

Now consider the FLRW universe with  $K = 0$ . We get from (44)

$$\rho_{d0} = \rho_d^c = \frac{3H_0^2}{8\pi G(1-\dot{\phi}^2)} \quad (47)$$

and the critical matter density becomes same as that of  $\rho_{d0}$ .

Now from (42) with  $K = 0$ , we have  $(\frac{\dot{a}}{a})^2 = \frac{C}{a^3}(1-\dot{\phi}^2)$ . We now take the negative square root of this equation so as to be consistent with observations. This will be borne out later. Therefore,

$$\frac{\dot{a}}{a} = -\frac{C^{\frac{1}{2}}}{a^{\frac{3}{2}}}(1-\dot{\phi}^2)^{\frac{1}{2}} \quad (48)$$

where  $C = \frac{8\pi G A}{3}$ . Using equations (20) and (48) we get

$$a(t) = C^{\frac{1}{3}} \left(\frac{\dot{\phi}}{\dot{\phi}_0}\right)^{\frac{2}{3}} (1-\dot{\phi}^2) \quad (49)$$

Therefore the deceleration parameter for non-relativistic case with  $K = 0$

$$q(t)^{NR} = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{Numerator}{Denominator}. \quad (50)$$

where,

$$\begin{aligned} Numerator = & (1-\dot{\phi}^2)[(1+20\dot{\phi}^2)(\ddot{\phi})^4 \\ & + \dot{\phi}\phi^{(3)}(\ddot{\phi})^2(1-4\dot{\phi}^2) \\ & + 5\dot{\phi}^2(\phi^{(3)})^2(-1+\dot{\phi}^2) - 3\dot{\phi}^2\ddot{\phi}\phi^{(4)}(-1+\dot{\phi}^2)] \end{aligned} \quad (51)$$

and

$$\begin{aligned} Denominator = & 2[(1-4\dot{\phi}^2)(\ddot{\phi})^2 + \dot{\phi}\phi^{(3)}(-1+\dot{\phi}^2)]^2 \end{aligned} \quad (52)$$

We shall take  $\phi^{(3)}, \phi^{(4)}$  to be zero also neglecting higher order of  $\dot{\phi}^2$ , where  $\phi^{(3)}$  is 3rd order derivative with respect to time. Then the deceleration parameter for non-relativistic case becomes,

$$q(t)^{NR} = \frac{1}{2}(1+27\dot{\phi}^2 + \dots) \quad (53)$$

Note that the choice of the sign of the square root (that leads to (48)) ensures that the value of the deceleration parameter for the matter dominated era is as in standard cosmology i.e. when dark energy is absent. This value is  $\frac{1}{2}$ . Moreover, it can be checked that choice of a positive square root (leading to (48)) will give an imaginary scale factor which is unacceptable.

### Relativistic case (Radiation dominated Universe)

For this case  $p_d = \frac{\rho_d}{3}$  from (40)  $\rho_d = \frac{B}{a^4}$  then modified Friedmann equations (35) and (36) becomes

$$\frac{H^2}{(1 - \dot{\phi}^2)} + \frac{K}{a^2} = \frac{8\pi G}{3} \frac{B}{a^4} \quad (54)$$

and

$$2\frac{\ddot{a}}{a} \frac{1}{(1 - \dot{\phi}^2)} + \frac{H^2}{(1 - \dot{\phi}^2)} + \frac{K}{a^2} + 2H \frac{\dot{\phi}\ddot{\phi}}{(1 - \dot{\phi}^2)^2} = -\frac{8\pi G}{3} \frac{B}{a^4} \quad (55)$$

Considering the  $K = 0$  model of the Universe, the modified Friedmann equation (54) becomes  $(\frac{\dot{a}}{a})^2 = \frac{D}{a^4}(1 - \dot{\phi}^2)$ . Again we take the negative square root of this equations from physical considerations to get

$$\frac{\dot{a}}{a} = -\frac{D^{\frac{1}{2}}}{a^2}(1 - \dot{\phi}^2)^{\frac{1}{2}} \quad (56)$$

where  $D = \frac{8\pi GB}{3}$ . Again combining equations (20) and (56) we obtain

$$a(t) = D^{\frac{1}{4}} \left(\frac{\dot{\phi}}{\dot{\phi}^3}\right)^{\frac{1}{2}} (1 - \dot{\phi}^2)^{\frac{3}{4}} \quad (57)$$

Therefore the deceleration parameter for relativistic case with  $K = 0$  is:

$$q(t)^R = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{\text{Numerator}}{\text{Denominator}} \quad (58)$$

where,

$$\begin{aligned} \text{Numerator} &= [(1 + 10\dot{\phi}^2 - 8\dot{\phi}^4)(\ddot{\phi})^4 \\ &- 3\dot{\phi}^2(\dot{\phi}^{(3)})^2(-1 + \dot{\phi}^2)^2 + 2\dot{\phi}^2(-1 + \dot{\phi}^2)^2\ddot{\phi}\dot{\phi}^{(4)}] \end{aligned} \quad (59)$$

and

$$\text{Denominator} = [(1 - 4\dot{\phi}^2)(\ddot{\phi})^2 + \dot{\phi}\dot{\phi}^{(3)}(-1 + \dot{\phi}^2)]^2 \quad (60)$$

Finally the deceleration parameter (neglecting as above higher order of  $\dot{\phi}^2$  and higher order derivatives) for relativistic case is

$$q(t)^R = 1 + 18\dot{\phi}^2 + \dots \quad (61)$$

Again, the choice of the sign of the square root (leading to (56)) ensures that the standard cosmology result is obtained for the deceleration parameter. This value is 1. Also choice of a positive square root is ruled out to ensure reality of the scale factor.

### Dark energy dominated Universe

For this case  $p_d \simeq -\rho_d$  and from (41)  $\rho_d = \text{constant} = W$ , then the modified Friedmann equations (35) and (36) becomes

$$\frac{\dot{a}^2}{a^2} \frac{1}{(1 - \dot{\phi}^2)} + \frac{K}{a^2} = \frac{8\pi G}{3} W \quad (62)$$

and

$$2\frac{\ddot{a}}{a} \frac{1}{(1 - \dot{\phi}^2)} + \frac{\dot{a}^2}{a^2} \frac{1}{(1 - \dot{\phi}^2)} + \frac{K}{a^2} + 2\frac{\dot{a}}{a} \frac{\dot{\phi}\ddot{\phi}}{(1 - \dot{\phi}^2)^2} = 8\pi GW \quad (63)$$

Again we consider  $K = 0$  model of the Universe, The modified Friedmann equation (62) becomes

$$\frac{\dot{a}}{a} = \alpha^{\frac{1}{2}}(1 - \dot{\phi}^2)^{\frac{1}{2}} \quad (64)$$

where  $\alpha = \frac{8\pi GW}{3} = \text{constant}$ . Now combining equations (20) and (64) we obtain;

$$\frac{\ddot{\phi}}{\dot{\phi}} = -\alpha^{\frac{1}{2}}(1 - \dot{\phi}^2)^{\frac{3}{2}}. \quad (65)$$

Now from equation (64) we get the scale factor

$$a(t) = e^{\sqrt{\alpha} \int \sqrt{1 - \dot{\phi}^2} dt} \quad (66)$$

Using above equation (66) we have the deceleration parameter for this case

$$q(t)^{\text{dark}} = -1 + \frac{\dot{\phi}\ddot{\phi}}{\sqrt{\alpha}(1 - \dot{\phi}^2)^{3/2}} \quad (67)$$

Further, using (65) the deceleration parameter becomes

$$q(t)^{\text{dark}} = -1 - \dot{\phi}^2 \quad (68)$$

As  $\dot{\phi}^2$  is positive the deceleration parameter is always negative.

Consider now an interesting situation. The dark energy density  $\dot{\phi}^2 < 1$  for reasons mentioned before. Then expanding the binomial in (65) and keeping terms upto  $O(\dot{\phi}^2)$ , an approximate solution for the dark energy density is obtained as

$$\dot{\phi}^2 = \frac{2\sqrt{\alpha}}{3\sqrt{\alpha} + 2e^{2\sqrt{\alpha}t}} \quad (69)$$

## CONCLUSION

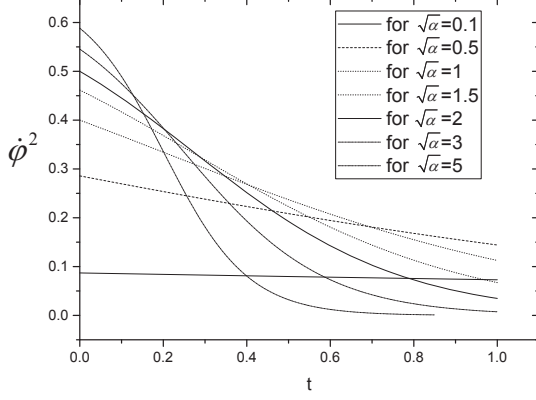


FIG. 1. Variation of dark energy density with time for positive  $\sqrt{\alpha}$  where values of  $\sqrt{\alpha}$  are shown down  $\rightarrow$  up.

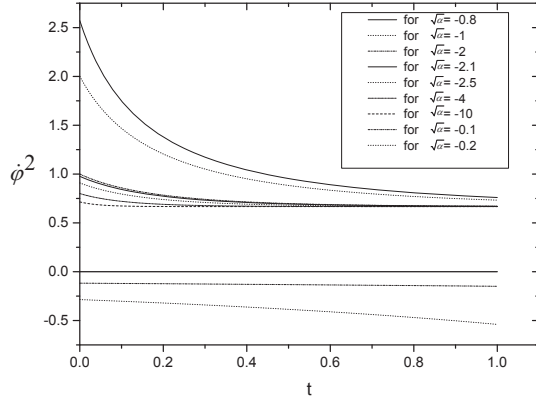


FIG. 2. Variation of dark energy density with time for negative  $\sqrt{\alpha}$  where values of  $\sqrt{\alpha}$  are shown up  $\rightarrow$  down.

Now  $\dot{\phi}^2 < 1$  applied to (69) means

$$\sqrt{\alpha} + 2e^{2\sqrt{\alpha}t} > 0 \quad (70)$$

Note that if  $\sqrt{\alpha}$  is positive then (70) is always satisfied for all values of  $t$ . However, figure 1 shows that there is absolutely no agreement of the predicted values of dark energy density with the observed data [28, 29] at present epoch. So we reject this choice.

On the other hand, taking the negative square root for  $\alpha$  gives encouraging agreement of predicted values for the dark energy density  $\dot{\phi}^2$  with the observed value at present epoch *viz.*, 0.6817 [28, 29]. This is evident in figure 2. So we choose the negative square root. The best agreements are obtained for  $-10 \leq \alpha \leq -2.1$ .

In this work we have investigated the cosmological consequences of incorporating dark energy in an emergent gravity scenario. First we obtained the analogues of the Friedman equations where the background metric is taken to be FLRW. Assuming the usual perfect fluid model for the universe, we next determined the total energy density. Finally, the cosmological implications were determined corresponding to various values of this energy. Our findings are as follows :

- (a) For total energy density greater than the pressure the deceleration parameter  $q(t) \approx \frac{1}{2}[1 + 27\dot{\phi}^2 + \dots] > \frac{1}{2}$ .
- (b) For total energy density equal to 3 times the pressure,  $q(t) \approx 1 + 18\dot{\phi}^2 + \dots > 1$  and
- (c) for total energy density equal to the negative of the pressure (dark energy scenario), the deceleration parameter  $q(t) < -1$ .

Note that for dark energy density  $\dot{\phi}^2 = 0$ , the conventional results are retrieved. Our results indicate that many aspects of standard cosmology can be accommodated with the presence of dark energy right from the beginning of the universe where the time parameter  $t \equiv \frac{t}{t_0}$ ,  $t_0$  being the present epoch.

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